AFRICA JOURNAL OF PHYSICAL SCIENCES

Africa Journal of Physical Sciences Vol. 3, pp. 55-66, February 2019
http://journals.uonbi.ac.ke/index.php/ajps/index
ISSN 2313-3317

Implications of a Late-Time Interaction in Vacuum Dark Energy Models

OKENG'O, Geoffrey [1,a]

[1]Department of Physics, University of Nairobi, P. O. Box 30197-00100 GPO Nairobi, Kenya

[a]gokengo@uonbi.ac.ke

ARTICLE INFO

Article History:
Submitted: 22 Jan 2018
Accepted: 13 Nov 2018
Available online: 28 February 2019

Keywords:

Dark energy,
dark sector interaction, vacuum energy,
energy-momentum transfer

ABSTRACT

Recently, Wang et al. (2014) and Salvatelli et al. (2014) have reported results showing that a late-time interaction between cold dark matter (CDM) and vacuum dark energy is favoured by current cosmological datasets and could alleviate the tension between the value of the Hubble constant obtained from Plank + WMAP polarisation data and that of the Hubble Space Telescope (HST). In this paper, we explore an alternative interacting vacuum dark energy model in which the net energy-momentum transfer is linear in the dark energy density and consider the behaviour for scalar perturbations in the conformal Newtonian gauge. Normalizing the energy densities to their present values, a positive interaction in the Friedmann Robertson Walker (FRW) background implies higher allowed values for the Hubble parameter, H_0, and the Dark Matter (DM) density than the in non-interacting model. Perturbations in the DM density deviate highly from the standard non-interacting case for higher interaction strengths regardless of direction of the energy transfer while in the linear DM power spectrum shows a shift degenerate with bias in the standard case.

ISSN 2313-3317, Vol. 3

1. Introduction

Explaining the source and nature of the observed late-time accelerated expansion of the universe remains one of the biggest challenges in modern cosmology today. Since its discovery over a decade ago using independent datasets from observations of supernovae Ia [3,4,5,6,7,8,9,10], Cosmic Microwave Background (CMB) anisotropy[11,12,13], Baryon Acoustic Oscillations (BAO)[14,15,16], Large Scale Structures (LSS)[17,18], weak gravitational lensing [19] and X-ray observations [20], many possible explanations have been put forth including dark energy and the infrared modification of Einstein's theory of gravity. Within the framework of general relativity, the possibilities that have be considered to explain such an acceleration include: dynamic dark energy with negative pressure $P = -\rho$ or scalar fields with a slow-rolling potential such as quintessence, quantum, phantom and even dark fluids.

The cosmological constant provides the simplest model that explains dark energy (DE) naturally without introducing additional dynamic degrees of freedom and fits well to the most recent cosmic microwave background data from *Planck*[22]. However, the vacuum model is notoriously plagued by two nagging problems: the "fine-tuning" problem- based on why the value of the DE density is very small today, and, the "coincidence" problem- why the DE density is currently of the same order to the DM density despite that their time evolution is very different[23,24].

An interaction between DE and DM provides an alternative way of alleviating the coincidence problem [25,26,27,28,29] and hence putting constraints on extensions to the standard ΛCDM model. Indeed, such an interaction is possible without violating existing observational constraints given our limited knowledge on dark sector physics[30,31]. Moreover, DE and DM being the dominant sources for the total cosmic energy budget, a zero interaction in the dark sector would be an approximation[2].

Studies of vacuum interacting dark energy (VIDE) have been shown recently to offer a variable explanation of the tension between the value of the Hubble constant H_0, from Planck+WMAP polarisation data and the HST measurements[32,33]. The study in[32] considers vacuum interactions with $Q \propto \rho_{dm}$ (ρ_{dm} is the dark matter density) and also work in the synchronous gauge. The study by [33] explores vacuum interaction of the form $[Q \propto H\rho_{de}]$ in the synchronous gauge, where H is the Hubble rate and ρ_{de} is the DE density.

In this paper, we consider the case of interacting vacuum DE with $Q \propto \rho_{de}$ and study perturbations in the conformal Newtonian gauge. The paper is arranged as follows: we describe general interacting dark energy background dynamics in section II and the perturbations in section III. In section IV we present the governing background and perturbation equations for our VIDE model. Our results and discussions are given in section V and the conclusions in section VI.

2. The Background Dynamics

A general dark-sector interaction in a FRW background universe can be described by the energy balance (non-conservation) equations,

$$\acute{\rho}_c + 3H\rho_c = -\acute{Q}. (1)$$

$$\acute{\rho}_x = \acute{Q}. \#(2)$$

Where the 'dot' denotes cosmic time derivative, 'm' stands for cold dark matter, 'x' stands for dark energy and $\acute{Q}$ is the background energy transfer rate. Here $H = \acute{a}/a$ is the Hubble parameter, which evolves according to the equation:

$$\acute{H} = -4\pi G\acute{\rho}_{tot}(1 + \omega_{tot}). (3)$$

Where $\omega_{tot} = \Omega_x\omega_x$.

Once a form for Q is chosen Equations(1), (2) and (3) completely describe the background dynamics for the VIDE model.

3. The Perturbations

A. Energy-momentum balance

The energy-momentum balance for the cold dark matter and dark energy fluids implies that,

$$\nabla_v T_c^{\mu v} = Q_c^{\mu}; \nabla_v T_x^{\mu v} = Q_x^{\mu}. (4)$$

Where T_{Av}^{μ} and Q_A^{μ} are the A-fluid energy-momentum tensor and 4-vectors respectively such total energy-momentum conservation fixes,

$$\sum Q_A^{\mu} = 0 \Longrightarrow Q_x^{\mu} = -Q_c^{\mu}. (5)$$

The A-fluid and total four-velocities are defined via,

$$u^{\mu} = a^{-1}\left(1 - \Phi, \partial^i v\right);$$

$$u_A^{\mu} = a^{-1}\left(1 - \Phi, \partial^i v\right). (6)\ our - velocities\ are\ defined\ as\ gauge.$$

A general energy-momentum four vector Q_A^{μ} can be split relative to the four-velocity u^{μ} as,

$$Q_A^{\mu} = Q_A u^{\mu} + F_A^{\mu}; Q_A = \acute{Q}_A + \delta Q_A. (7)$$

Where $F_A^{\mu} = a^{-1}(0, \partial^{\iota} f_A)$ is the momentum density transfer rate relative to u^{μ} to first order, f_A is the gauge-invariant momentum transfer potential and $u_{\mu} F_A^{\mu} = 0$. Combining equations (7) and (6) gives,

$$Q_0^A = -a\left[\acute{Q}_A(1 + \Phi) + \delta Q_A\right]. (8)$$

$$Q_i^A = a\partial_i\left[\acute{Q}_A(v + B) + f_A\right]. (9)$$

And hence equations(4),(7), (8) and (9) give the density and velocity perturbation equations[2,34,35],

$$\delta_A' + 3H(c_{sA}^2 - \omega_A)\delta_A - k^2(1 + \omega_A)v_A - 9H(1 + \omega_A)(c_{sA}^2 - c_{aA}^2)v_A - 3(1 + \omega_A)\Phi' = \frac{a\acute{Q}_A}{\acute{\rho}_A}[\Phi - \delta_A - 3H(c_{sA}^2 - c_{aA}^2)v_A]\frac{a\delta Q_A}{\acute{\rho}_A}. (10)$$

$$v_A'(1 + \omega_A) + H(1 - 3c_{sA}^2)(1 + \omega_A)v_A + c_{sA}^2\delta_A + (1 + \omega_A)\Phi = \frac{a}{\acute{\rho}_A}\{\acute{Q}_A[v - (1 + c_{sA}^2)v_A]f_A\}. (11)$$

Here 'curl H' is the Hubble rate divided by the scale factor, w_A is the A-fluid equation of state parameter and c_s is the speed of sound.

B. Field equations

The perturbed Friedmann-Robertson- Walker (FRW) line element in the conformal Newtonian gauge is,

$$ds^2 = a^2\left[-(1+2\Phi)d\eta^2 + (1-2\Phi)\delta_{ij}dx^i dx^j\right]. (12)$$

Where $\Phi$the gravitational potential and η is is the conformal time.

The $0-i$ and the $0-0$ components of the Einstein Field Equations (EFEs) yield,

$$\Phi' = -4\pi G a^2 \sum_A W_A - H\Phi\,. (13)$$

$$k^2\Phi = -4\pi G a^2\left(\sum_A \rho\delta_A - 3H\sum_A W_A\right). (14)$$

Where $W_A = (1+\omega_A)\rho_A v_A$ and,

$$\delta_A = \Delta_A - \frac{\acute{\rho}'_A}{\acute{\rho}_A} v_A. (15)$$

4. Our Vacuum Vide Model

A. Relativistic Poisson equation

For the ΓVIDE model the relativistic Poisson equation (14) has the form,

$$k^2\Phi = \frac{-3}{2}H^2(\Omega_c\delta_c + \Omega_x\delta_x). (16)$$

With the interacting effects coming only through the DM and DE density contrasts which by equation (15) are,

$$\delta_c = \Delta_c + \left(H + a\Gamma\frac{\Omega_x}{\Omega_c}\right)v_c. (17)$$

$$\delta_x = \Delta_x - a\Gamma v_{x.}(18)$$

B. Perturbed energy-momentum treatment

For the simplest physical choice for the energy-momentum transfer described by vanishing momentum transfer either in the DM or the DE frame, we obtain two sub-classes with the energy-momentum transfer four-vectors parallel to the DM or the DE frame. We denote these sub-classes by $Q\|u_c$ and $Q\|u_x$ respectively.

For adiabatic DM fluid, we have$c_{sC}^2 = c_{aC}^2 = \omega_c = 0$. However, the DE fluid is non-adiabatic with $c_{ax}^2 = \omega_x = -1$ and $c_{sx}^2 = 1$ where the latter corresponds to the physical sound speed for quintessence (a self-consistent model for DE), chosen to avoid physical instabilities.

A commonly used variable choice of the covariant energy-momentum four-vector, Q^{μ} is[2,34,35],

$$Q^{\mu} = \Gamma \rho_x u^{\mu}. (19)$$

Where in the interacting VIDE background model the energy transfer rate is,

$$\acute{Q} = \Gamma \acute{\rho}_x. (20)$$

And for the perturbations we have,

$$Q = \Gamma \rho_x = \Gamma \acute{\rho}_x (1 + \delta_x). (21)$$

For the $Q \| u_c$ model the evolution equations become,

$$\delta_c' - k^2 v_c - 3\Phi' = a\Gamma \frac{\Omega_x}{\Omega_c}(\delta_c - \delta_x - \Phi). (22)$$

$$v'_x - 3H\left(v_c - \frac{4}{3}v_x\right) = -\Phi. (23)$$

$$v'_c + Hv_c = -\Phi. (24)$$

$$\Phi' + H\Phi = \frac{-3}{2}H^2 \Omega_c v_c. (25)$$

Additionally, from the Euler equation (11) and equation (15) we obtain respectively,

$$\delta_x = a\Gamma(v_c - 2v_x). (26)$$

$$\Delta_x = a\Gamma(v_c - v_x). (27)$$

Subtracting equation (24) from (23) we obtain,

$$(v_x - v_c)' = -4H(v_x - v_c). (28)$$

Which implies that,

$$v_x - v_c = (v_{xd} - v_{cd})\left(\frac{a_d}{a}\right)^4. (29)$$

So that for an adiabatic DM fluid equation (29) implies that we have $v_x = v_c$ which gives,

$$\delta_x = -a\Gamma v_x. (30)$$

$$\Delta_x = 0. (31)$$

Equation (30) implies that the DE fluid is non-adiabatic while equation (31) tells us that the DE perturbations vanish in the comoving DM frame which is quite unnatural given that in this model the energy-momentum transfer is along the DM 4-velocity.

For the $Q \| u_x$ model, the evolution equations set are,

$$\delta'_c - k^2 v_c - 3\Phi' = a\Gamma \frac{\Omega_x}{\Omega_c}(\delta_c - \delta_x - \Phi)(32)$$

$$v'_x + Hv_x = -\Phi(33)$$

$$v'_c - a\Gamma \frac{\Omega_x}{\Omega_c}(v_c - v_x) + Hv_c = -\Phi(34)$$

$$\Phi' + H\Phi = \frac{-3}{2}H^2 \Omega_c v_c(35)$$

From the Euler equation (11) and equation (15) we obtain,

$$\delta_x = -a\Gamma v_x (36)$$

$$\Delta_x = 0 (37)$$

Subtracting equation (34) from equation (33) we obtain,

$$(v_x - v_c)' = -\left(H - a\Gamma \frac{\Omega_x}{\Omega_c}\right)(v_x - v_c) (38)$$

In a similar manner to the $Q\|u_c$ case, if the DM fluid is non-adiabatic equation (29) implies that$v_c = v_x$, hence for both $Q\|u_c$ and $Q\|u_x$ cases we have only two independent DM density and velocity evolution equations and the closed set of equations become,

$$\delta'_c - k^2 v_c - 3\Phi' = a\Gamma \frac{\Omega_x}{\Omega_c}(\delta_c - \delta_x - \Phi) (39)$$

$$v'_c + Hv_c = -\Phi (40)$$

$$\Phi' + H\Phi = \frac{-3}{2} H^2 \Omega_c v_c (41)$$

Where the DE perturbation I Newtonian gauge is $\delta_x = -a\gamma v_c$ by equation (30) or (36).

We note that putting $\Gamma = 0$ in equation (39) recovers the fiducial (non-interacting) ΛCDM model described by the well-known equations,

$$\delta'_c - k^2 v_c - 3\Phi' = 0 (42)$$

$$v'_c + Hv_c = -\Phi (43)$$

$$\Phi' + H\Phi = \frac{-3}{2} H^2 \Omega_c v_c (44)$$

C. Initial conditions

At decoupling we have,

- DM peculiar velocity:

$$\Phi' = 0 \Longrightarrow v_{cd} = \frac{-2\Phi_d}{3H_d\Omega_{cd}} (45)$$

- DM density contrast:

$$\Delta_{cd} = \frac{-2k^2}{3H_d^2\Omega_{cd}} \Phi_d (46)$$

- DE density contrast:

$$\delta_{xd} = -a\Gamma v_{xd} (47)$$

Where equation (47) means that we can have either $\delta_{xd} = 0$ or $v_{xd} = 0$ but not both.

- Gravitational potential:

$$\Phi_d(k) = \frac{9}{10} T(k) \Phi_p(k) (48)$$

Where,

$$\Phi_p(k) = A \frac{\Omega_{c0}}{D_\Phi(k, a=0)} \left(\frac{k}{H_0}\right)^{(n-4)/2} (49) \rightarrow (49)$$

The constant A fixes the primordial amplitude of curvature perturbations and $n = 0.96$ is the scalar spectroscopic index.

D. Growth Functions and Power Spectrum

The growth functions are defined by [35],

$$D_{\Phi}(k,a) = \frac{\Phi(k,a)}{\Phi_d(k)} a (50)$$

$$D_A(k,a) = \frac{\Delta_A(k,a)}{\Delta_{Ad}(k)} a_d (51)$$

$$D_{uA}(k,a) = \frac{u_A(k,a)}{u_{Ad}} \alpha_A (52)$$

Where α_A is a constant whose value for the ΛCDM peculiar velocity is,

$$\alpha_A = \frac{2la_d^2 E_d}{3\Omega_{c0}} (53)$$

The matter power spectrum is given by [36],

$$P_c = \frac{9A^2}{50\pi^3} l^2 T(k)^2 \left[\frac{D_c(k,a)}{D_{\Phi}(k,a=1)}\right]^2 (54)$$

5. Results and Discussions

By normalizing the energy densities and the Hubble rate to their values today,$\Omega_{c0} = 0.3, \Omega_{x0} = 0.7$ and $H_0 = 72$, the background equations describing the interaction can be solved numerically. The ΓVIDE model predicts a higher DM density (Lower DE density) in the past than for the non-interacting ΛCDM case for$\gamma > 0$ i.e. energy transfer from DM to DE. This reverses when the transfer of energy sis from DE to DM for $\gamma < 0$ as shown in figure 1. A positive γ increases the value of the allowed Hubble constant in the ΓVIDE model, converging to the same value of H_0 due to our normalisation as displayed in figure 2. We find that the Hubble rate for the VIDE model is more sensitive to energy transfer from DE to DM and deviates strongly from the standard case for more negative values of the interaction parameter. The higher value of the Hubble rate at early times predicted by the VIDE model means that the luminosity and angular diameter distances from our model are larger than the standard case. This effect is shown in figure (3).

The effect of the vacuum interaction on the growth rates of DM perturbations is shown in figure 4. The results show that when the transfer of energy is from DM to DE $(\gamma > 0)$ there will be higher growth rates for the DM perturbations than when the transfer if from DE to DM$(\gamma < 0)$. The high growth rates are driven by the normalisation of the DM density that leads to a higher value of Ω_c at early times than in the non-interacting case.

It is also apparent that the transfer of energy from DE to DM suppresses the growth of density perturbations as also pointed out by [37] and[2]. Plots for the $Q\|u_c$ case with higher growth rates at late times of energy transfer from DM to DE.

The matter power spectrum in the universe today shows a shift from the standard ΛCDM case at all scales driven by the equality of the DM and DE peculiar velocities for adiabatic perturbations. A plot of the linear matter power spectrum and the ratio of the DM to the gravitational potential growth rates are shown in figure 6 for the$Q\|u_c$.

6. Conclusions

We have presented from a study of an interaction between a cold DM fluid and a DE fluid assumed to be a cosmological constant, and investigated the cosmological implications of the interaction scenario, comparing this to the standard ΛCDM case when adiabatic initial conditions are used. By re-writing the general perturbation equations and casting them into a suitable form, we have eliminated a singularity reported in many previous studied (see e.g. [34] and [2] thus accommodating the case when $\omega = -1$ not previously investigated. Starting from the perturbation equations in Newtonian gauge for our interaction model, we have derived simple dimensionless D-function ordinary differential equations that eliminate the problem of computing the perturbation initial conditions at decoupling. Our equations provide a simple platform on which to investigate interacting models with constant equation of state parameter and forms a useful benchmark for studies aimed at clearing some existing subtleties in interacting vacuum dark energy studies.

We have studied variable energy-momentum treatment that considers the energy-momentum 4-vector as linear to the dark energy density and obtained results showing that:

- An interaction between DM and DE modifies the background evolution leading to deviations that can be large for stronger interaction strengths.
- Normalising the DM and DE densities to their values today, the resulting shape of density profile is dependent on the direction of the energy transfer and can accommodate large interaction values as pointed out in [2].
- By re-writing the perturbation equations in a more suitable and consistent form singularities in the Euler perturbation equations reported in see e.g., [2,34] can be eliminated.
- For scalar perturbations in Newtonian gauge, our VIDE model shows growth of the matter density and curvature perturbations that become more visible after the onset of the DE domination and at late-times and over all scales if adiabatic initial conditions are used.
- The choice of adiabatic initial conditions for the VIDE model leads to similar evolution of DM density perturbations independent of the two energy-momentum sub-classes characterised by the 4-vector along the DM 4-velocity or the DE $-velocity.
- The linear matter power spectrum displays a shift at all scales but approaches the standard ΛCDM matter spectrum in the limit of vanishing interaction.
- Our model results are void of the instabilities reported previously for constant *w* interacting dark energy models.

Acknowledgments

This work was supported by the South African National Research Foundation (NRF) through the South African Square Kilometre Array (SKA) project grant. GO was supported by the University of Nairobi (UoN) study leave.

References

[1.] Amendola, L., and S. Tsujikawa, (2010) Dark Energy: Theory and Observations: Cambridge, Cambridge University Press, 485 p.

[2.] Clemson, T., K. Koyama, G.-B. Zhao, R. Maartens, and J. Väliviita, (2012) Interacting dark energy: Constraints and degeneracies: Physical Review D, **85**(4): 043007, doi:10.1103/PhysRevD.85.043007.

[3.] Perlmutter, S., Aldering, G., Valle, M.D., Deustua, S., Ellis, R.S., Fabbro, S., Fruchter, A., Goldhaber, G., Groom, D.E., Hook, I.M., Kim, A.G., Kim, M.Y., Knop, R.A., Lidman, C., McMahon, R.G., Nugent, P., Pain, R., Panagia, N., Pennypacker, C.R., Ruiz-Lapuente, P., Schaefer, B., and Walton, N., (1998) Discovery of a supernova explosion at half the age of the Universe. Nature **391**:51–54, https://doi.org/10.1038/34124

[4.] Perlmutter, S., M. S. Turner, and M. White, (1999) Constraining Dark Energy with Type Ia Supernovae and Large-Scale Structure: Physical Review Letters, **83**(4):670–673, doi:10.1103/PhysRevLett.83.670.

[5.] Perlmutter, S., Aldering, G., Deustua, S., Fabbro, S., Goldhaber, G., Groom, D.E., Kim, A.G., Kim, M.Y., Knop, R.A., Nugent, P., Pennypacker, C.R., Goobar, A., Pain, R., Hook, I.M., Lidman, C., Ellis, R.S., Irwin, M., McMahon, R.G., Ruiz-Lapuente, P., Walton, N., Schaefer, B., Boyle, B.J., Filippenko, A.V., Matheson, T., Fruchter, A.S., Panagia, N., Newberg, H.J.M., and Couch, W.J., (1998) Cosmology from Type Ia Supernovae. Bulletin of the American Astronomical Society, **29**:1351

[6.] Riess, A.G., Filippenko, A.V., Challis, P., Clocchiatti, A., Diercks, A., Garnavich, P.M., Gilliland, R.L., Hogan, C.J., Jha, S., Kirshner, R.P., Leibundgut, B., Phillips, M.M., Reiss, D., Schmidt, B.P., Schommer, R.A., Smith, R.C., Spyromilio, J., Christopher Stubbs, Suntzeff, N.B., and Tonry, J. (1998) Observational Evidence from Supernovae for an Accelerating Universe and a Cosmological Constant. The Astronomical Journal **116**(3): 1009, https://doi.org/10.1086/300499

[7.] Adam G. Riess, (2000) The Case for an Accelerating Universe from Supernovae: Publications of the Astronomical Society of the Pacific, **112**(776):1284.

[8.] Suzuki, N., Rubin, D., Lidman, C., Aldering, G., Amanullah, R., Barbary, K., Barrientos, L.F., Botyanszki, J., M. Brodwin, Connolly, N., Dawson, K.S., Dey, A., Doi, M., Donahue, M., Deustua, S., Eisenhardt, P., Ellingson, E., L. Faccioli, Fadeyev, V., Fakhouri, H.K., Fruchter, A.S., Gilbank, D.G., Gladders, M.D., Goldhaber, G., Gonzalez, A.H., Goobar, A., Gude, A., Hattori, T., Hoekstra, H., Hsiao, E., Huang, X., Ihara, Y., Jee, M.J., D. Johnston, Kashikawa, N., Koester, B., Konishi, K., Kowalski, M., Linder, E.V., Lubin, L., Melbourne, J., J. Meyers, Morokuma, T., Munshi, F., Mullis, C., Oda, T., Panagia, N., Perlmutter, S., Postman, M., Pritchard, T., J. Rhodes, Ripoche, P., Rosati, P., Schlegel, D.J., Spadafora, A., Stanford, S.A., Stanishev, V., Stern, D., M. Strovink, Takanashi, N., Tokita, K., Wagner, M., Wang, L., Yasuda, N., Yee, H.K.C., and Project, T.S.C., (2012) The Hubble Space Telescope Cluster Supernova Survey. V. Improving the Dark-energy Constraints above z > 1 and Building an Early-type-hosted Supernova Sample. The Astrophysical Journal, **746**:85. https://doi.org/10.1088/0004-637X/746/1/85

[9.] Tonry, J.L., Schmidt, B.P., Barris, B., Candia, P., Challis, P., Clocchiatti, A., Coil, A.L., Filippenko, A.V., Garnavich, P., Hogan, C., Holland, S.T., Jha, S., Kirshner, R.P., Krisciunas, K., Leibundgut, B., Li, W., Matheson, T., Phillips, M.M., Riess, A.G., Schommer, R., Smith, R.C., Sollerman, J., Spyromilio, J., Stubbs, C.W., and Suntzeff, N.B., (2003) Cosmological Results from High-z Supernovae. The Astrophysical Journal, **594**(1):1. https://doi.org/10.1086/376865

[10.] Tegmark, M., Strauss, M.A., Bahcall, N.A., Schlegel, D., Finkbeiner, D., Gunn, J.E., Ostriker, J.P., Seljak, U., Ivezic, Z., Knapp, G.R., Lupton, R.H., Blanton, M.R., Scoccimarro, R., Hogg, D.W., Abazajian, K., Yongzhong, X., Dodelson, S., Sandvik, H., Xiaomin, W., and Jain, B., (2004) Cosmological parameters from SDSS and WMAP. Physical Review. D, Particles Fields, **69**:103501-103501.26, https://doi.org/10.1103/PhysRevD.69.103501

[11.] Gold, B., Odegard, N., Weiland, J.L., Hill, R.S., Kogut, A., Bennett, C.L., Hinshaw, G., Chen, X., Dunkley, J., M. Halpern, Jarosik, N., Komatsu, E., Larson, D., Limon, M., Meyer, S.S., Nolta, M.R., Page, L., Smith, K.M., Spergel, D.N., Tucker, G.S., Wollack, E., and Wright, E.L., (2011) Seven-year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Galactic Foreground Emission. The Astrophysical Journal, **192**:15, https://doi.org/10.1088/0067-0049/192/2/15

[12.] Komatsu, E., Smith, K.M., Dunkley, J., Bennett, C.L., Gold, B., Hinshaw, G., Jarosik, N., Larson, D., Nolta, M.R., Page, L., Spergel, D.N., Halpern, M., Hill, R.S., Kogut, A., Limon, M., Meyer, S.S., Odegard, N., Tucker, G.S., Weiland, J.L., Wollack, E., and Wright, E.L., (2011) Seven-year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Cosmological Interpretation. The Astrophysical Journal, **192**:18, https://doi.org/10.1088/0067-0049/192/2/18

[13.] Hinshaw, G., Larson, D., Komatsu, E., Spergel, D.N., Bennett, C.L., Dunkley, J., Nolta, M.R., Halpern, M., Hill, R.S., Odegard, N., Page, L., Smith, K.M., Weiland, J.L., Gold, B., Jarosik, N., Kogut, A., Limon, M., Meyer, S.S., Tucker, G.S., Wollack, E., and Wright, E.L., (2013) Nine-year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Cosmological Parameter Results. The Astrophysical Journal **208**:19, https://doi.org/10.1088/0067-0049/208/2/19

[14.] Beutler, F., Blake, C., Colless, M., Jones, D.H., Staveley-Smith, L., Campbell, L., Parker, Q., Saunders, W., and Watson, F., (2011) The 6dF Galaxy Survey: baryon acoustic oscillations and the local Hubble constant. Monthly Notices of the Royal Astronomical Society **416**:3017–3032. https://doi.org/10.1111/j.1365-2966.2011.19250.x

[15.] Percival, W.J., Reid, B.A., Eisenstein, D.J., Bahcall, N.A., Budavari, T., Frieman, J.A., Fukugita, M., Gunn, J.E., Ivezić, Ž., Knapp, G.R., Kron, R.G., Loveday, J., Lupton, R.H., McKay, T.A., Meiksin, A., Nichol, R.C., Pope, A.C., Schlegel, D.J., Schneider, D.P., Spergel, D.N., Stoughton, C., Strauss, M.A., Szalay, A.S., Tegmark, M., Vogeley, M.S., Weinberg, D.H., York, D.G., and Zehavi, I., (2010) Baryon acoustic oscillations in the Sloan Digital Sky Survey Data Release 7 galaxy sample. Monthly Notices of the Royal Astronomical Society **401**:2148–2168, https://doi.org/10.1111/j.1365-2966.2009.15812.x

[16.] Eisenstein, D.J., Zehavi, I., Hogg, D.W., Scoccimarro, R., Blanton, M.R., Nichol, R.C., Scranton, R., Seo, H.-J., Tegmark, M., Zheng, Z., Anderson, S.F., Annis, J., Neta Bahcall, Brinkmann, J., Burles, S., Castander, F.J., Connolly, A., Csabai, I., Mamoru Doi, Fukugita, M., Frieman, J.A., Glazebrook, K., Gunn, J.E., Hendry, J.S., Gregory Hennessy, Ivezić, Z., Kent, S., Knapp, G.R., Lin, H., Loh, Y.-S., Lupton, R.H., Bruce Margon, McKay, T.A., Meiksin, A., Munn, J.A., Pope, A., Richmond, M.W., David Schlegel, Schneider, D.P., Shimasaku, K., Stoughton, C., Strauss, M.A., Mark SubbaRao, Szalay, A.S., Szapudi, I., Tucker, D.L., Yanny, B., and York, D.G., (2005) Detection of the Baryon Acoustic Peak in the Large-Scale Correlation Function of SDSS Luminous Red Galaxies. The Astronomical Journal **633**(2):560, https://doi.org/10.1086/466512

[17.] Percival, W.J., Baugh, C.M., Bland-Hawthorn, J., Bridges, T., Cannon, R., Cole, S., Colless, M., Collins, C., Couch, W., Dalton, G., De Propris, R., Driver, S.P., Efstathiou, G., Ellis, R.S., Frenk, C.S., Glazebrook, K., Jackson, C., Lahav, O., Lewis, I., Lumsden, S., Maddox, S., Moody, S., Norberg, P., Peacock, J.A., Peterson, B.A., Sutherland, W., and Taylor, K., (2001) The 2dF Galaxy Redshift Survey: The power spectrum and the matter content of the universe. Monthly Notices of the Royal Astronomical Society **327**(4):1297–1306, https://doi.org/10.1046/j.1365-8711.2001.04827.x

[18.] Reid, B.A., Percival, W.J., Eisenstein, D.J., Verde, L., Spergel, D.N., Skibba, R.A., Bahcall, N.A., Budavari, T., Frieman, J.A., Fukugita, M., Gott, J.R., Gunn, J.E., Ivezić, Ž., Knapp, G.R., Kron, R.G., Lupton, R.H., McKay, T.A., Meiksin, A., Nichol, R.C., Pope, A.C., Schlegel, D.J., Schneider, D.P., Stoughton, C., Strauss, M.A., Szalay, A.S., Tegmark, M., Vogeley, M.S., Weinberg, D.H., York, D.G., and Zehavi, I., (2010) Cosmological constraints from the clustering of the Sloan Digital Sky Survey DR7 luminous red galaxies. Monthly Notices of the Royal Astronomical Society **404**:60–85, https://doi.org/10.1111/j.1365-2966.2010.16276.x

[19.] Suyu, S.H., Marshall, P.J., Auger, M.W., Hilbert, S., Blandford, R.D., Koopmans, L.V.E., Fassnacht, C.D., and Treu, T., (2010) Dissecting the Gravitational lens B1608+656. II. Precision Measurements of the Hubble Constant, Spatial Curvature, and the Dark Energy Equation of State. The Astrophysical Journal, **711**:201, https://doi.org/10.1088/0004-637X/711/1/201

[20.] Allen, S.W., Schmidt, R.W., Ebeling, H., Fabian, A.C., and van Speybroeck, L., (2004. Constraints on dark energy from Chandra observations of the largest relaxed galaxy clusters. Monthly Notices of the Royal Astronomical Society **353**:457–467. https://doi.org/10.1111/j.1365-2966.2004.08080.x

[21.] R. Durrer and R. Maartens. *Dark Energy and Modified Gravity* (2010) "Dark Energy: Observational & Theoretical Approaches", ed. P Ruiz-Lapuente (Cambridge UP, 2010), pp48 - 91.

[22.] Planck Collaboration, Ade, P.A.R., Aghanim, N., Armitage-Caplan, C., Arnaud, M., Ashdown, M., Atrio-Barandela, F., Aumont, J., Baccigalupi, C., Banday, A.J., Barreiro, R.B., Bartlett, J.G., Battaner, E., Benabed, K., Benoît, A., Benoit-Lévy, A., Bernard, J.-P., Bersanelli, M., Bielewicz, P., Bobin, J., Bock, J.J., Bonaldi, A., Bond, J.R., Borrill, J., Bouchet, F.R., Bridges, M., Bucher, M., Burigana, C., Butler, R.C., Calabrese, E., Cappellini, B., Cardoso, J.-F., Catalano, A., Challinor, A., Chamballu, A., Chary, R.-R., Chen, X., Chiang, H.C., Chiang, L.-Y., Christensen, P.R., Church, S., Clements, D.L., Colombi, S., Colombo, L.P.L., Couchot, F., Coulais, A., Crill, B.P., Curto, A., Cuttaia, F., Danese, L., Davies, R.D., Davis, R.J., de Bernardis, P., de Rosa, A., de Zotti, G., Delabrouille, J., Delouis, J.-M., Désert, F.-X., Dickinson, C., Diego, J.M., Dolag, K., Dole, H., Donzelli, S., Doré, O., Douspis, M., Dunkley, J., Dupac, X., Efstathiou, G., Elsner, F., Enßlin, T.A., Eriksen, H.K., Finelli, F., Forni, O., Frailis, M., Fraisse, A.A., Franceschi, E., Gaier, T.C., Galeotta, S., Galli, S., Ganga, K., Giard, M., Giardino, G., Giraud-Héraud, Y., Gjerløw, E., González-Nuevo, J., Górski, K.M., Gratton, S., Gregorio, A., Gruppuso, A., Gudmundsson, J.E., Haissinski, J., Hamann, J., Hansen, F.K., Hanson, D., Harrison, D., Henrot-Versillé, S., Hernández-Monteagudo, C., Herranz, D., Hildebrandt, S.R., Hivon, E., Hobson, M., Holmes, W.A., Hornstrup, A., Hou, Z., Hovest, W., Huffenberger, K.M., Jaffe, A.H., Jaffe, T.R., Jewell, J., Jones, W.C., Juvela, M., Keihänen, E., Keskitalo, R., Kisner, T.S., Kneissl, R., Knoche, J., Knox, L., Kunz, M., Kurki-Suonio, H., Lagache, G., Lähteenmäki, A., Lamarre, J.-M., Lasenby, A., Lattanzi, M., Laureijs, R.J., Lawrence, C.R., Leach, S., Leahy, J.P., Leonardi, R., León-Tavares, J., Lesgourgues, J., Lewis, A., Liguori, M., Lilje, P.B., Linden-Vørnle, M., López-Caniego, M., Lubin, P.M., Macías-Pérez, J.F., Maffei, B., Maino, D., Mandolesi, N., Maris, M., Marshall, D.J., Martin, P.G., Martínez-González, E., Masi, S., Massardi, M., Matarrese, S., Matthai, F., Mazzotta, P., Meinhold, P.R., Melchiorri, A., Melin, J.-B., Mendes, L., Menegoni, E., Mennella, A., Migliaccio, M., Millea, M., Mitra, S., Miville-Deschênes, M.-A., Moneti, A., Montier, L., Morgante, G., Mortlock, D., Moss, A., Munshi, D., Murphy, J.A., Naselsky, P., Nati, F., Natoli, P., Netterfield, C.B., Nørgaard-Nielsen, H.U., Noviello, F., Novikov, D., Novikov, I., O'Dwyer, I.J., Osborne, S., Oxborrow, C.A., Paci, F., Pagano, L., Pajot, F., Paoletti, D., Partridge, B., Pasian, F., Patanchon, G., Pearson, D., Pearson, T.J., Peiris, H.V., Perdereau, O., Perotto, L., Perrotta, F., Pettorino, V., Piacentini, F., Piat, M., Pierpaoli, E., Pietrobon, D., Plaszczynski, S., Platania, P., Pointecouteau, E., Polenta, G., Ponthieu, N., Popa, L., Poutanen, T., Pratt, G.W., Prézeau, G., Prunet, S., Puget, J.-L., Rachen, J.P., Reach, W.T., Rebolo, R., Reinecke, M., Remazeilles, M., Renault, C., Ricciardi, S., Riller, T., Ristorcelli, I., Rocha, G., Rosset, C., Roudier, G., Rowan-Robinson, M., Rubiño-Martín, J.A., Rusholme, B., Sandri, M., Santos, D., Savelainen, M., Savini, G., Scott, D., Seiffert, M.D., Shellard, E.P.S., Spencer, L.D., Starck, J.-L., Stolyarov, V., Stompor, R., Sudiwala, R., Sunyaev, R., Sureau, F., Sutton, D., Suur-Uski, A.-S., Sygnet, J.-F., Tauber, J.A., Tavagnacco, D., Terenzi, L., Toffolatti, L., Tomasi, M., Tristram, M., Tucci, M., Tuovinen, J., Türler, M., Umana, G., Valenziano, L., Valiviita, J., Van Tent, B., Vielva, P., Villa, F., Vittorio, N., Wade, L.A., Wandelt, B.D., Wehus, I.K., White, M., White, S.D.M., Wilkinson, A., Yvon, D., Zacchei, A., and Zonca, A., (2014) Planck 2013 results. XVI. Cosmological parameters. Astronomy & Astrophysics **571**:A16. https://doi.org/10.1051/0004-6361/201321591

[23.] Böhmer, C.G., Caldera-Cabral, G., Lazkoz, R., and Maartens, R., (2008) Dynamics of dark energy with a coupling to dark matter. Phys. Rev. **D 78**:023505. https://doi.org/10.1103/PhysRevD.78.023505

[24.] Copeland, E.J., Sami, M., and Tsujikawa, S., (2006) Dynamics of dark energy. Int. J. Mod. Phys. **D 15**:1753–1935. https://doi.org/10.1142/S021827180600942X

[25.] Copeland, E.J., Liddle, A.R., and Wands, D., (1998) Exponential potentials and cosmological scaling solutions. Phys. Rev. **D 57**:4686–4690. https://doi.org/10.1103/PhysRevD.57.4686

[26.] Amendola, L., (1999) Scaling solutions in general nonminimal coupling theories. Phys. Rev. **D60**:043501. https://doi.org/10.1103/PhysRevD.60.043501

[27.] Amendola, L., (2000) Coupled quintessence. Phys. Rev. **D 62**:043511. https://doi.org/10.1103/PhysRevD.62.043511

[28.] Guo, Z.-K., Cai, R.-G., and Zhang, Y.Z., (2005) Cosmological evolution of interacting phantom energy with dark matter. J. Cosmol. Astropart. Phys. **2005**:002. https://doi.org/10.1088/1475-7516/2005/05/002

[29.] Amendola L., Quercellini C., Tocchini-Valentini, D., and Pasqui A., (2003) Constraints on the interaction of dark energy from cosmic microwave background. Astrophys. J.,**583**: L53

[30.] H. Wei. (2011) Cosmological Constraints on the Sign-Changeable Interactions. Commun. Theor. Phys., **56**:972-980.

[31.] Xia J. Q., (2009) Constraints on coupled dark energy models from observations. Phys. Rev., **D80**: 103514.

[32.] Wang Y., Wands D., Zhao G.-B., and Xu L., (2014) *Post-Planck Constraints on interacting vacuum energy*. Phys. Rev. **D 90**:023502.

[33.] Salvatelli V., Said N., Bruni M., Melchiorri A., and Wands D., (2014) Indications of a late-time interaction in the dark sector. Phys. Rev. Lett. **113**:181301.

[34.] Valiviita J., Majerotto E., and Maartens R., (2008) Instability in interacting dark energy and dark matter fluids. JCAP**, 0807**: 020.

[35.] Hashim M., Bertacca D., and Maartens R., (2014) Degeneracy between primordial non-Gaussianity and interaction in the dark sector. Phys. Rev. **D 90**:103518.

[36.] Duniya D., Bertacca D., and Maartens R., (2013) Clustering of quintessence on horizon scales and its imprint on HI intensity mapping. JCAP, **1310**:015.

[37.] Caldera-Caral G., Maartens R., and Schaefer B. M., (2009) *The Growth of Structure in Interacting Dark Energy Model*. JCAP, **0907**:027.